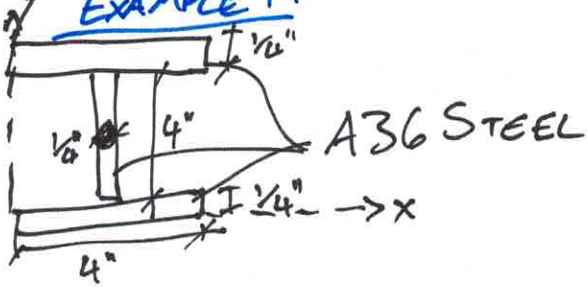
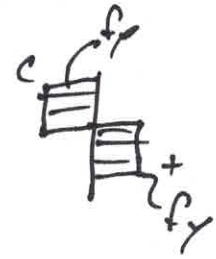
Elastic Vs Plastic Neutral Axis

Example 1!





E. N. A.

$$\bar{y}_e \sum_i A_i = \sum_i y_i A_i$$

$$\bar{y}_e = \frac{\sum y_i A_i}{\sum_i A_i}$$

$$\bar{y}_e = \frac{.125'' \times 1/4'' \times 4'' + 2.25'' \times 1in^2 + 4.375'' \times 1in^2}{1in^2 + 1in^2 + 1in^2}$$

$$\bar{y}_e = \frac{.125in^3 + 2.25in^3 + 4.375in^3}{3in^2}$$

$$\bar{y}_e = \frac{6.75in^3}{3in^2} \Rightarrow \boxed{\bar{y}_e = 2.25in}$$

P. N. A.

$$\sum_i \cancel{F_{y,i\,above}} \cdot A_{above} = \sum_i \cancel{F_{y,i\,below}} \cdot A_{i\,below}$$

$$\sum_i A_{above} = \sum A_{below}$$

☑ Assume Vertical Plate holds PNA

$$1/4'' \times 4'' + 1/4''(4.25'' - \bar{y}_p) = 1/4'' \times 4'' + 1/4(\bar{y}_p - .$$

$$1in^2 + 1/4''(4.25'' - \bar{y}_p) = 1in^2 + 1/4(\bar{y}_p - .2$$

$$-1in^2 \qquad /1/4'' \qquad -1in^2 \qquad /1/4''$$

$$4.25'' - \bar{y}_p = \bar{y}_p - .25''$$

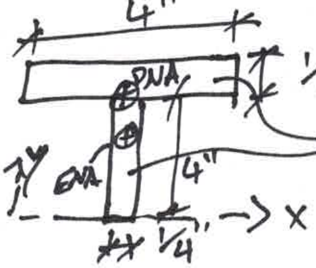
$$\Rightarrow \quad 4.5'' = 2 \cdot \bar{y}_p$$

$$\boxed{\bar{y}_p = 2.25''}$$

$$0.25'' \le \bar{y}_p \le 4.25''$$

Assumption ☑

EXAMPLE 2



A36 Steel

E.N.A.:

$$\bar{y}_e = \frac{1/4'' \times 4'' \times 2'' + 4'' \times 1/4'' \times 4.125''}{1\,in^2 + 1\,in^2}$$

$$\bar{y}_e = \frac{2\,in^3 + 4.125''}{2\,in^2}$$

$$\boxed{\bar{y}_e = 3.0625''}$$

P.N.A.:

$$\sum \cancel{F_y} A_{above} = \sum \cancel{F_{y\,below}} A_{below}$$

☑ Assume $0 \le \bar{y}_p \le 4''$

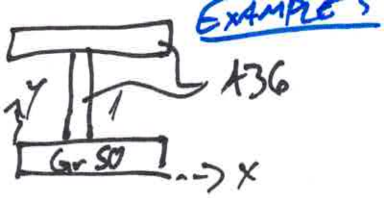
$$1/4'' \times \bar{y}_p = (4'' - \bar{y}_p)\,1/4'' + 1/4'' \times 4''$$

$$0.25''\,\bar{y}_p = 1\,in^2 - 0.25''\,\bar{y}_p + 1\,in^2$$

$$0.5''\,\bar{y}_p = 2\,in^2$$

$$\boxed{\bar{y}_p = 4\,in}$$

Assumption: $0 \le \bar{y} \le 4''$ ☑



(3) $\frac{1}{4}" \times 4"$ Steel Plates.

$E_{A36} = 29{,}000\ ksi$ $\quad F_{y_{A36}} = 36\ ksi$

$E_{Gr50} = 29{,}000\ ksi$ $\quad F_{y_{Gr50}} = 50\ ksi$

$$\boxed{n = \frac{29{,}000}{29{,}000} = 1}$$

ENA:

$$\boxed{\bar{y}_e = 2.25"}$$

P.N.A.:

$$\Sigma F_{y_{above}} A_{above} = \Sigma (F_y A)_{below}$$

☐ Assume $0.25" \leq \bar{y}_p \leq 4.25"$

$$36ksi \cdot 4" \times \tfrac{1}{4}" + 36ksi \cdot \tfrac{1}{4}" \cdot (4.25" - \bar{y}_p) = 50ksi \cdot 4" \cdot \tfrac{1}{4}" + 36ksi \cdot \tfrac{1}{4}"$$

$$36\ \overset{kips}{\cancel{ksi \cdot in^2}} + 9\ kip/in \cdot (4.25" - \bar{y}_p) = 50k + 9\ kips/in(\bar{y}_p - 0.25")$$

$$-36\ kips \quad -9\ k/in(\bar{y}_p - 0.25") \qquad \cancel{50k}\ -36k \quad -9\ k/in(\bar{y}_p - 0.25")$$

$$\frac{9k/in(4.5" - 2\bar{y}_p)}{9\ k/in} = \frac{14k}{9\ k/in}$$

$$4.5" - 2\bar{y}_p = \frac{14}{9}\ in$$

$$-4.5" \qquad -4.5"$$

$$\frac{-2\bar{y}_p}{-2} = \frac{\frac{14}{9}\ in - 4.5"}{-2}$$

$$\boxed{\bar{y}_p = 1.472\ in}$$

$0.25" \leq \bar{y}_p < 4.25"$ ☑

Assumption Valid!

$(\Sigma A_{above} \cdot F_y)$ or $(\Sigma A_{below} F_y)$ $\cdot$ $A_{\text{center of } A_{above}\ \&\ A_{below}}$ $= M_p$